Unit 3 : Faraday’s law and Maxwell’s equations

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Electromagnetic Induction

A changing magnetic field (intensity, movement) will induce an electromotive force (emf)

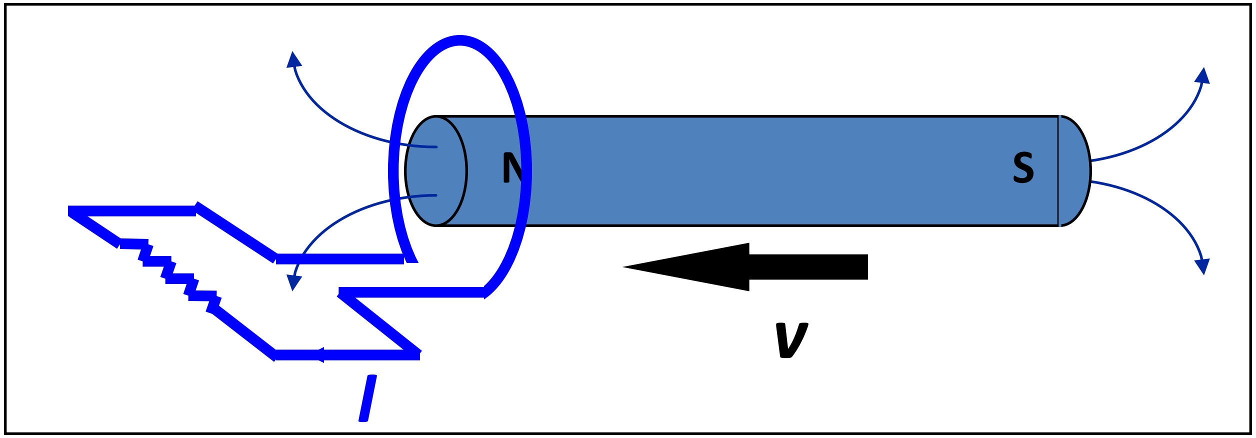
In a closed electric circuit, a changing magnetic field will produce an electric current

Electromagnetic Induction
Faraday's Law

The induced emf in a circuit is proportional to the rate of change of magnetic flux, through any surface bounded by that circuit.

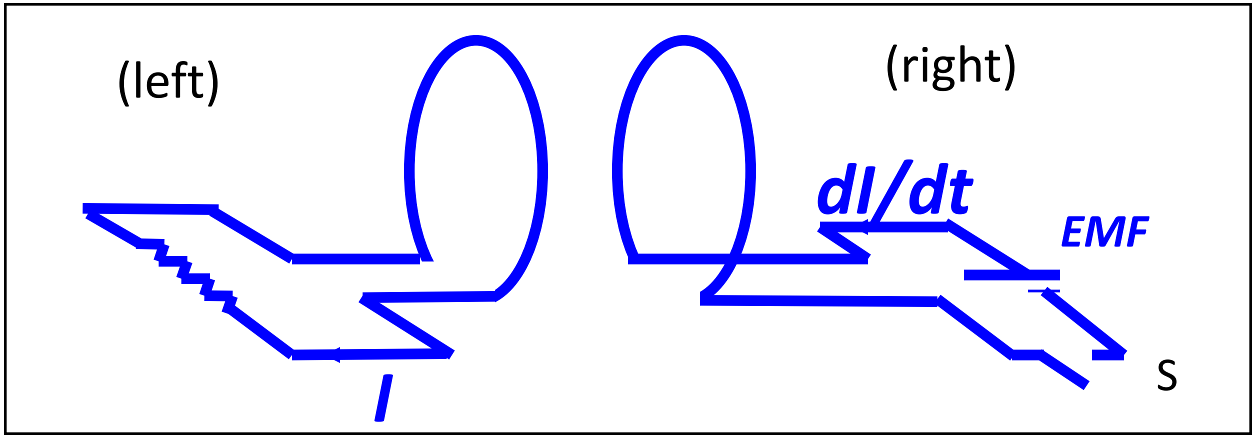
$$\boxed{\mathcal{E} = - d\Phi_B / dt}$$

Faraday's Experiments



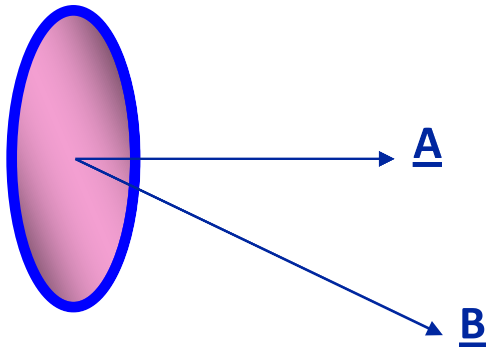
- Michael Faraday discovered induction in 1831.
- Moving the magnet induces a current I.
- Reversing the direction reverses the current.
- Moving the loop induces a current.
- The induced current is set up by an *induced EMF.*

Faraday’s Experiments



- Changing the current in the right-hand coil induces a current in the left-hand coil.
- The induced current does not depend on the size of the current in the right-hand coil.
- The induced current depends on ***dI/dt***.

Magnetic Flux

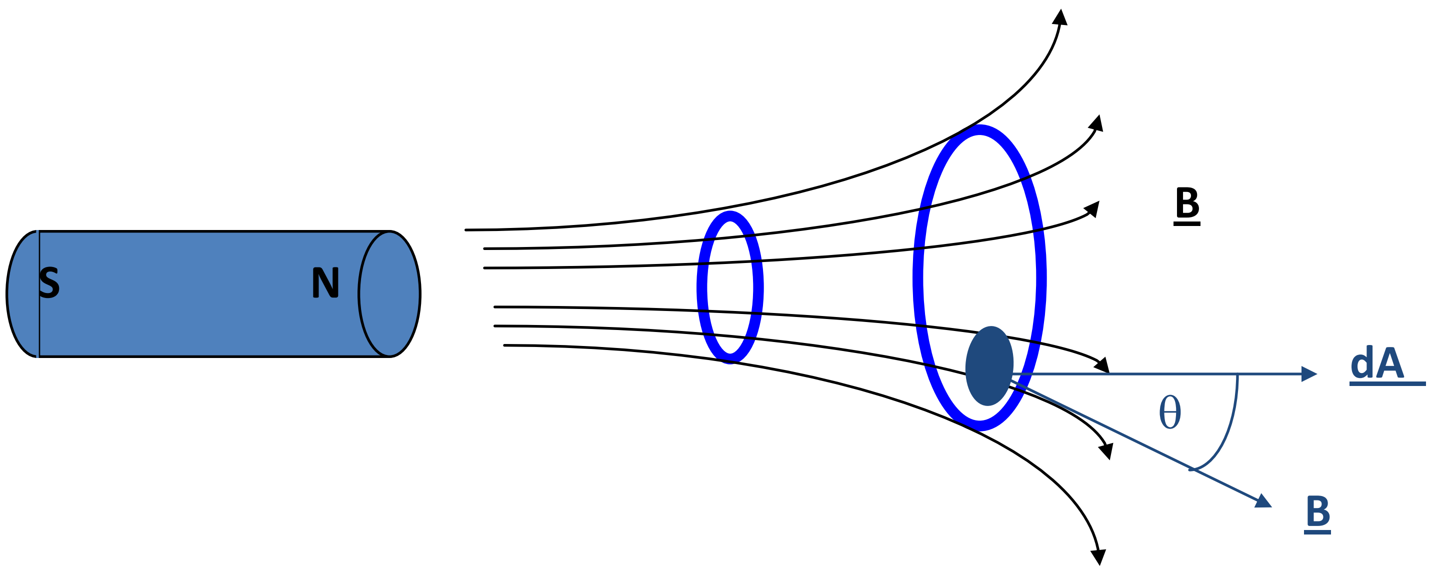


- In the easiest case, with a constant magnetic field $\underline{\mathbf{B}}$, and a flat surface of area $\mathbf{A}$, the magnetic flux is

$$\boxed{\Phi_B = \underline{\mathbf{B}} \cdot \underline{\mathbf{A}}}$$

- Units : 1 tesla x m^2 = 1 weber

Magnetic Flux

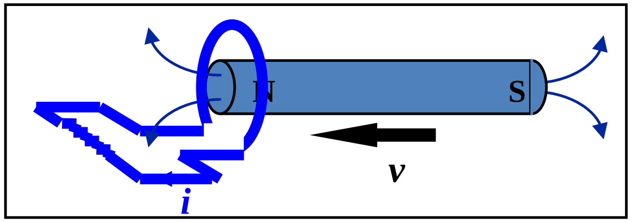


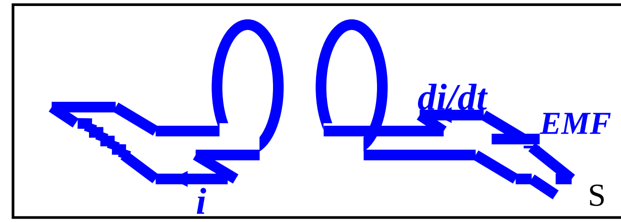
- When B is not constant, or the surface is not flat, one must do an integral.
- Break the surface into bits **dA**. The flux through one bit is

$$d\Phi_B = \mathbf{B} \cdot d\mathbf{A} = B\, dA \cos\theta.$$

- Add the bits:

Faraday's Law

1) 

2) 

- Moving the magnet changes the flux Φ_B (1).
- Changing the current changes the flux Φ_B (2).
- ***Faraday:*** changing the flux induces an emf.

$$\mathcal{E} = - d\Phi_B /dt$$

Faraday's law

The emf induced around a loop

equals the rate of change of the flux through that loop

Lenz's Law

- Faraday's law gives the direction of the induced emf and therefore the direction of any induced current.

- Lenz's law is a simple way to get the directions straight, with less effort.

- **Lenz's Law:**

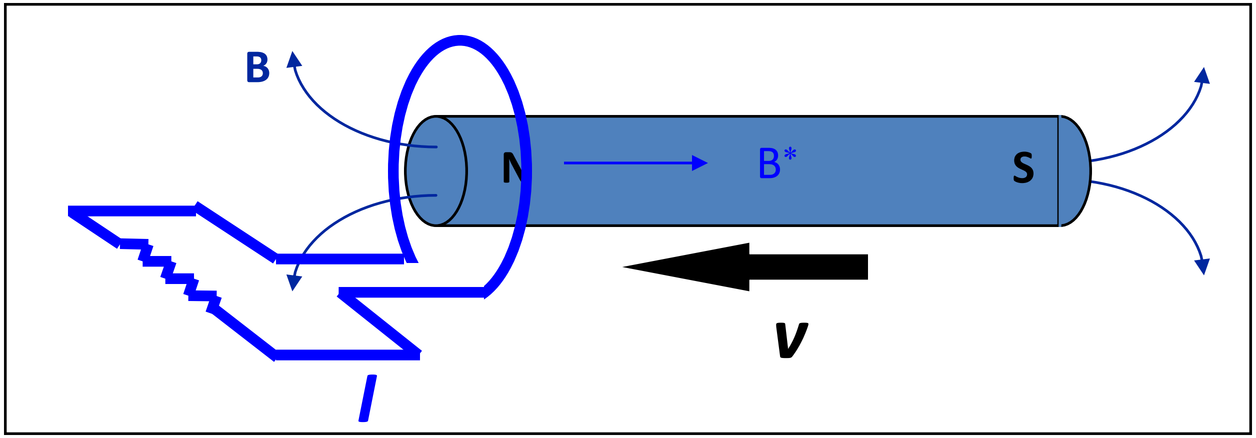
  The induced emf is directed so that any induced current flow will *oppose* the *change* in magnetic flux (which causes the induced emf).

- This is easier to use than to say ...

  Decreasing magnetic flux $\Rightarrow$ emf creates additional magnetic field

  Increasing flux $\Rightarrow$ emf creates opposed magnetic field

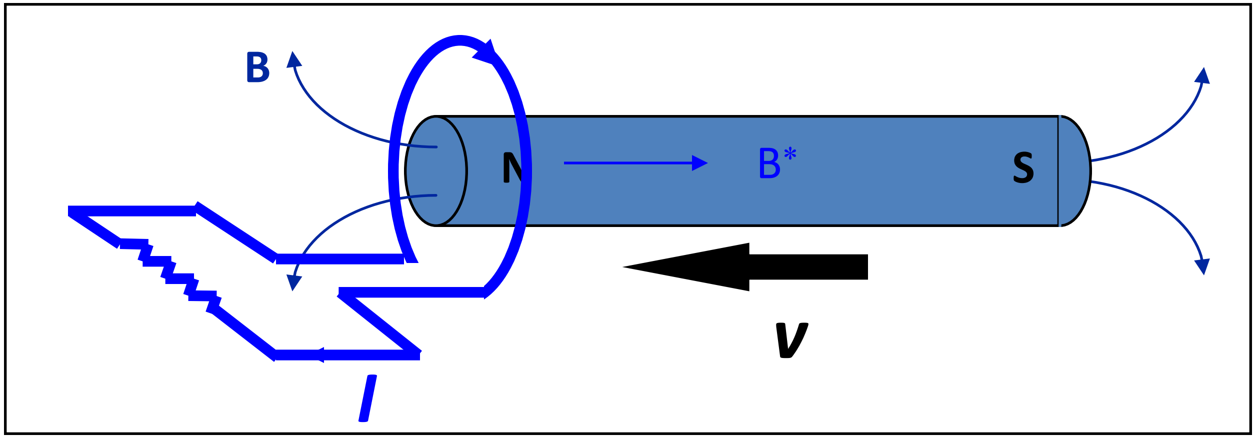
Lenz's Law



If we move the magnet towards the loop the flux of B will increase.

Lenz's Law $\Rightarrow$ the current induced in the loop will generate a field B* opposed to B.

Lenz's Law

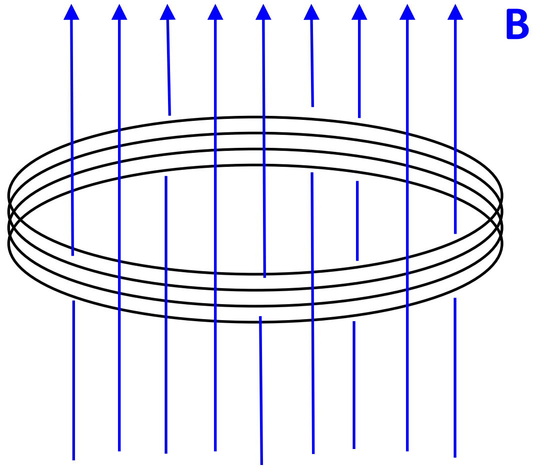


If we move the magnet towards the loop the flux of B will increase.

Lenz's Law $\Rightarrow$ the current induced in the loop will generate a field B* opposed to B.

Example of Faraday's Law

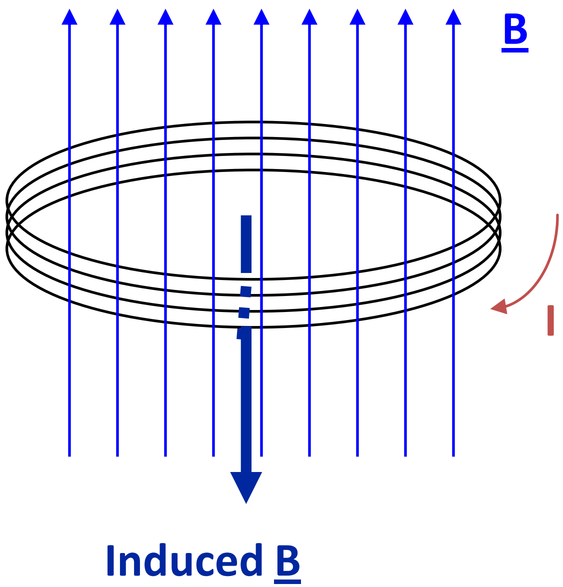
Consider a coil of radius 5 cm with N = 250 turns.
A magnetic field B, passing through it,
changes in time: B(t)= 0.6 t [T] (t = time in seconds)
The total resistance of the coil is 8 Ω.
What is the induced current ?



Use Lenz's law to determine the direction of the induced current.

Apply Faraday's law to find the emf and then the current.

Example of Faraday's Law



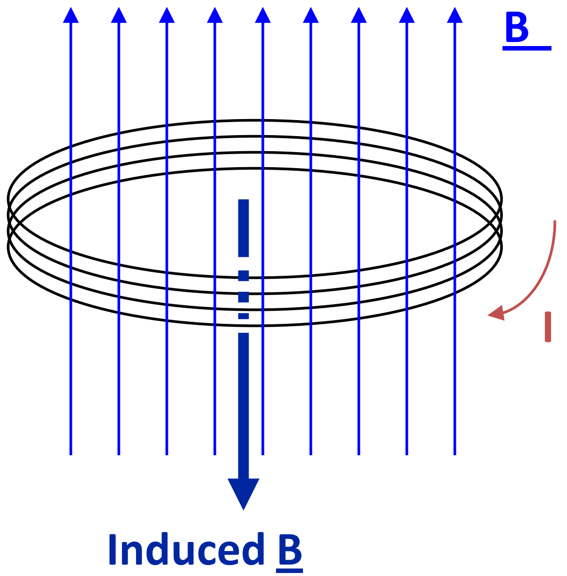
Lenz's law:

The change in B is increasing the upward flux through the coil.

So the induced current will have a magnetic field whose flux (and therefore field) are ***down***.

Hence the induced current must be ***clockwise*** when looked at from above.

Use Faraday's law to get the magnitude of the induced emf and current.



The induced EMF is $\mathcal{E} = - d\Phi_B /dt$

Here $\Phi_B = N(BA) = NB(\pi r^2)$

Therefore $\mathcal{E} = - N(\pi r^2)\, dB/dt$

Since $B(t) = 0.6t$, $dB/dt = 0.6$ T/s

Thus

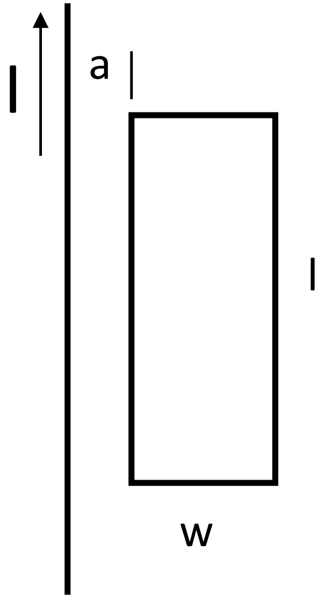
$\mathcal{E} = -(250)(\pi\, 0.005^2)(0.6T/s) = -1.18$ V $\quad (1V=1Tm^2/s)$

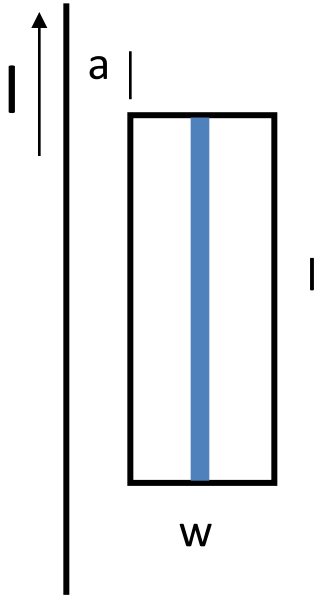
Current $I = \mathcal{E} / R = (-1.18V) / (8\,\Omega) = -0.147$ A

It's better to ignore the sign and get directions from Lenz's law.

Magnetic Flux in a Nonuniform Field

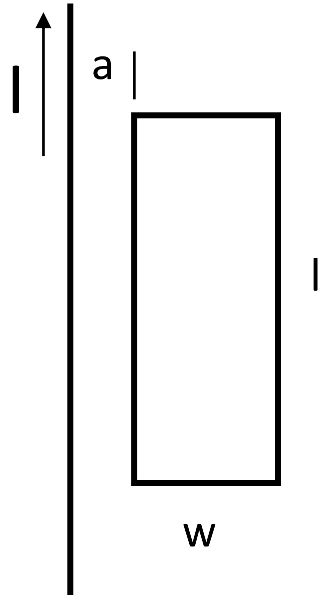
A long, straight wire carries a current I. A rectangular loop (w by l) lies at a distance a, as shown in the figure. What is the magnetic flux through the loop?.





Induced emf Due to Changing Current

A long, straight wire carries a current $I = I_0 + \alpha t$.
A rectangular loop (w by l) lies at a distance a,
as shown in the figure.
What is the induced emf in the loop?.
What is the direction of the induced current and field?

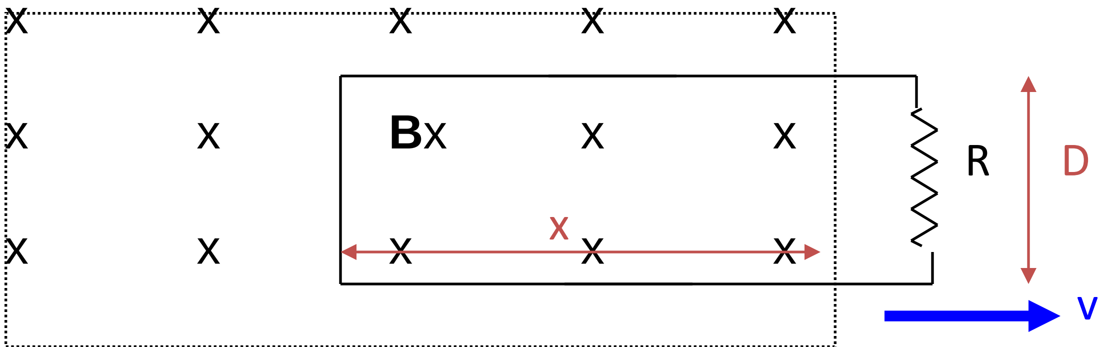


Motional EMF

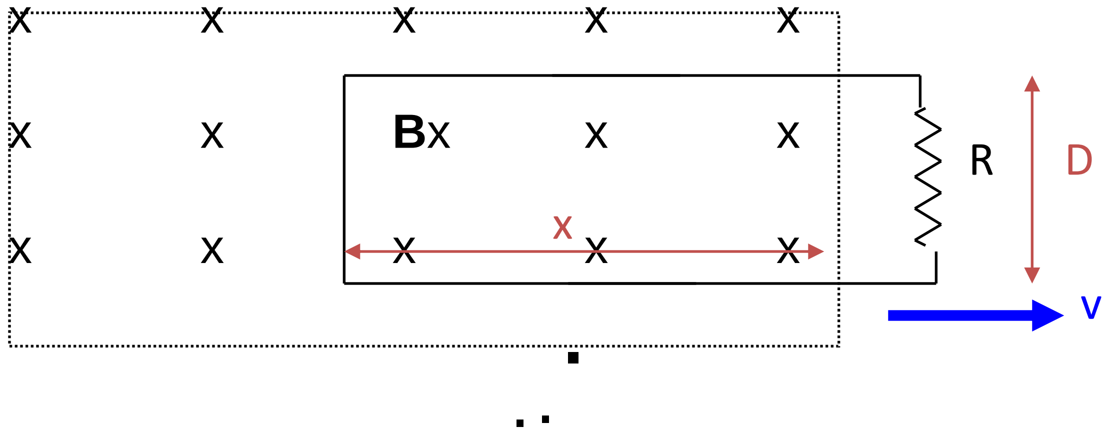
Up until now we have considered fixed loops.
The flux through them changed because the magnetic field changed with time.

Now try moving the loop in a uniform and constant magnetic field. This changes the flux, too.

<u>B</u> points into screen



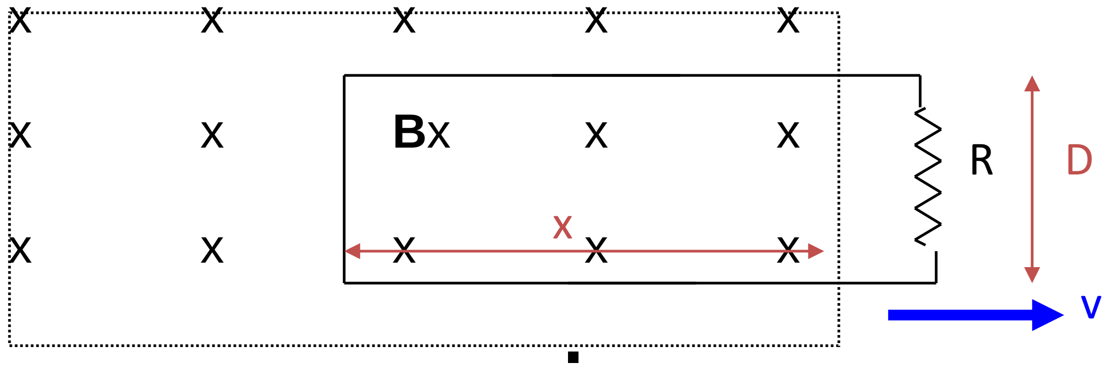
Motional EMF - Use Faraday's Law



The flux is $\Phi_B = \underline{B}\ \underline{A} = BDx$

This changes in time:

Motional EMF - Use Faraday's Law

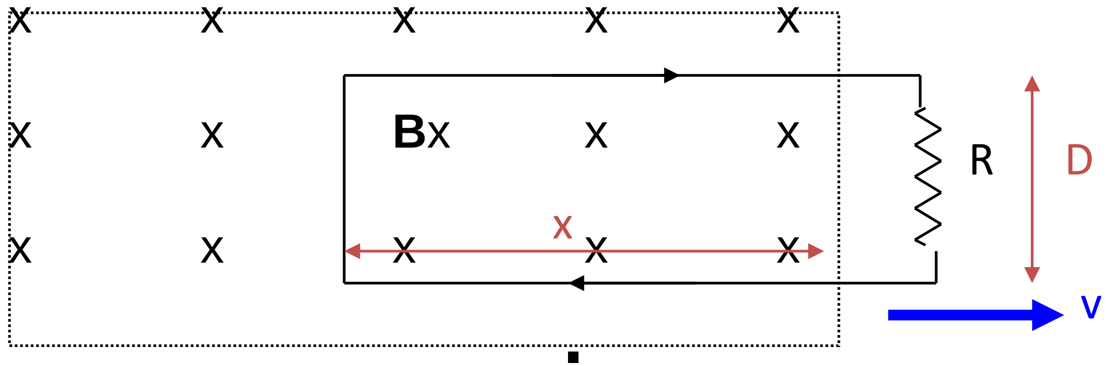


The flux is $\Phi_B = \underline{B}\ \underline{A} = BDx$

This changes in time:

$$d\Phi_B / dt = d(BDx)/dt = BDdx/dt = -BDv$$

Hence by Faraday's law there is an induced emf and current. What is the direction of the current?



The flux is $\Phi_B = \underline{B}\ \underline{A} = BDx$

This changes in time:

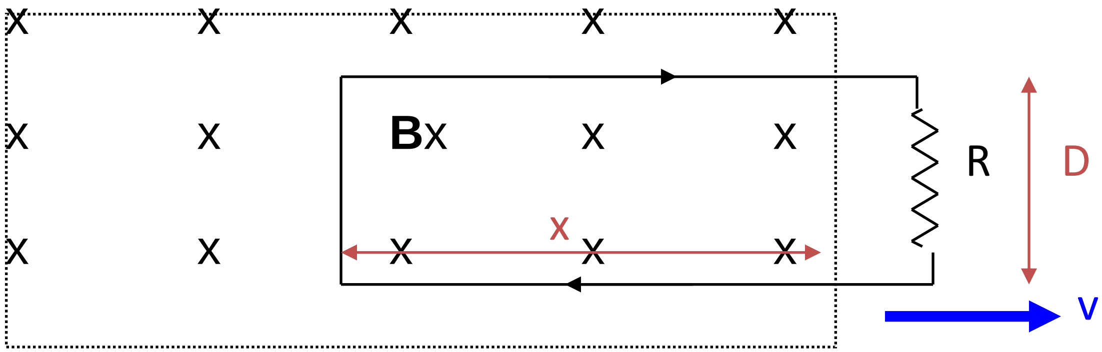
$$d\Phi_B / dt = d(BDx)/dt = BDdx/dt = -BDv$$

Hence by Faraday's law there is an induced emf and current. What is the direction of the current?

Lenz's law: there is less inward flux through the loop. Hence the induced current gives inward flux.

$\Rightarrow$ So the induced current is clockwise.

Motional EMF
Faraday's Law



Now Faraday's Law $\mathcal{E} = -d\Phi_B/dt$

gives the EMF $\Rightarrow \mathcal{E} = BDv$

In a circuit with a resistor, this gives

$$\mathcal{E} = BDv = IR \Rightarrow I = BDv/R$$

Thus moving a circuit in a magnetic field produces an emf exactly like a battery. This is the principle of an electric generator.

Maxwell's Equations of Electromagnetism

Gauss' Law for Electrostatics

$$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0}$$

Gauss' Law for Magnetism

$$\oint \underline{B} \bullet \underline{dA} = 0$$

Faraday's Law of Induction

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

Ampere's Law

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

Consider these equations in a vacuum.....

......no mass, no charges. no currents.....

$$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0} \quad \longrightarrow \quad \oint \underline{E} \bullet \underline{dA} = 0$$

$$\oint \underline{B} \bullet \underline{dA} = 0 \quad \longrightarrow \quad \oint \underline{B} \bullet \underline{dA} = 0$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt} \quad \longrightarrow \quad \oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \longrightarrow \quad \oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

$$\boxed{\oint \underline{E} \bullet \underline{dA} = 0}$$

$$\boxed{\oint \underline{B} \bullet \underline{dA} = 0}$$

$$\boxed{\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}}$$

$$\boxed{\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

Energy in Electromagnetic Waves

Energy density in matter for static fields

$$\frac{dU}{dv} = \frac{1}{2}\left(\mathbf{D.E} + \mathbf{B.H}\right)$$

In vacuum $\mathbf{D} = \varepsilon_o \mathbf{E}$ $\mathbf{B} = \mu_o \mathbf{H}$ $\varepsilon_r = \mu_r = 1$

$$\mathbf{E} = \mathbf{E}_o \exp^{i(\omega t - kz)} \quad \mathbf{H} = \mathbf{H}_o \exp^{i(\omega t - kz)}$$

$$\frac{dU}{dv} = \frac{1}{2}\left(\varepsilon_o \mathbf{E.E} + \mu_o \mathbf{H.H}\right)$$

$$= \frac{1}{2}\left(\varepsilon_o \mathbf{E}_o.\mathbf{E}_o + \mu_o \mathbf{H}_o.\mathbf{H}_o\right)\cos^2(\omega t - kz)$$

$$\frac{d\overline{U}}{dv} = \frac{1}{4}\left(\varepsilon_o \mathbf{E}_o.\mathbf{E}_o + \mu_o \mathbf{H}_o.\mathbf{H}_o\right) \qquad \left\langle \cos^2(\omega t - kz) \right\rangle = \frac{1}{2}$$

Average obtained over one cycle of light wave

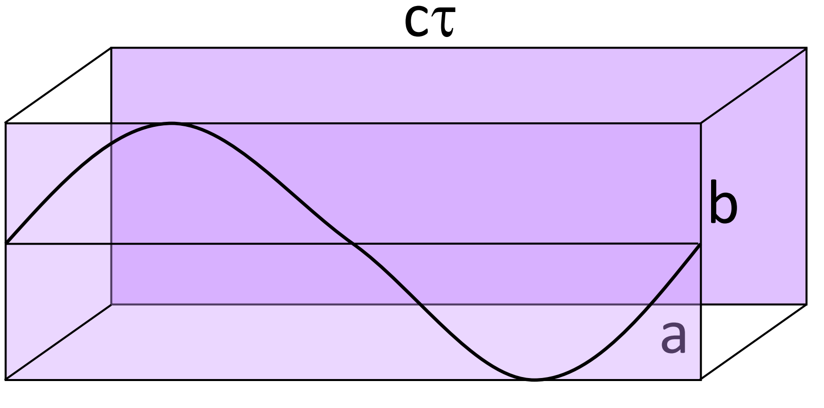
Energy in Electromagnetic Waves

Average energy over one cycle of light wave

$$\frac{d\overline{U}}{dv} = \frac{1}{4}\left(\varepsilon_o \mathbf{E}_o . \mathbf{E}_o + \mu_o \mathbf{H}_o . \mathbf{H}_o\right)$$

Distance travelled by light over one cycle = $2\pi c/\omega$ = $c\tau$

Average energy in volume ab $c\tau$



Energy in Electromagnetic Waves

$$\overline{\mathrm{U}} = \frac{1}{4}\left(\varepsilon_o \mathbf{E}_\mathrm{o}.\mathbf{E}_\mathrm{o} + \mu_o \mathbf{H}_\mathrm{o}.\mathbf{H}_\mathrm{o}\right)\mathrm{abc}\tau$$

$$\mathrm{H_o} = \frac{\mathrm{B_o}}{\mu_o} \qquad \mathrm{c} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \qquad \mathrm{B_o} = \frac{\mathrm{E_o}}{\mathrm{c}}$$

$$\frac{\overline{\mathrm{U}}}{\mathrm{abc}\tau} = \frac{1}{2}\mathrm{E_o H_o}\left(\frac{1}{2}\frac{\mathrm{E_o}}{\mathrm{H_o}}\varepsilon_o + \frac{1}{2}\frac{\mathrm{H_o}}{\mathrm{E_o}}\mu_o\right)$$

$$\frac{\mathrm{E_o}}{\mathrm{H_o}} = \frac{\mathrm{B_o c}}{\mathrm{B_o}}\mu_o = \frac{\mu_o}{\sqrt{\mu_o \varepsilon_o}} = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

$$\frac{\overline{\mathrm{U}}}{\mathrm{abc}\tau} = \frac{1}{2}\mathrm{E_o H_o}\left(\frac{1}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\varepsilon_o + \frac{1}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\mu_o\right) = \frac{1}{2\mathrm{c}}\mathrm{E_o H_o}$$

$$\frac{\overline{\mathrm{U}}}{\mathrm{ab}\tau} = \frac{1}{2}\mathrm{E_o H_o}$$ Energy crossing unit area (ab) per periodic time (τ)

Poynting Vector

N = **E** x **H** is the Poynting vector

Equal to the instantaneous energy flow associated with an EM wave

In vacuum **N** || wave vector **k**

Example If the **E** amplitude of a plane wave is 0.1 Vm^{-1}
Energy crossing unit area per second is

$$\frac{1}{2}E_o H_o = \frac{1}{2}E_o^2\sqrt{\frac{\mu_o}{\varepsilon_o}} = 1.3.10^{-5}\ Wm^{-2}$$

Future Scope and relevance to industry

- https://www.researchgate.net/publication/295291761_Applications_of_Faraday's_Laws_of_Electrolysis_in_Metal_Finishing
- http://iopscience.iop.org/article/10.1088/0143-0807/33/3/L15
- http://iopscience.iop.org/article/10.1088/0143-0807/33/2/397